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10MCA12

First Semester MCA Degree Examination, June/July 2013

Discrete Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Define power set of a set A. IF A = {a, ϕ} then find the power set, P(A). Also prove that if a set A has n elements then P(A) has 2^n elements. **(05 Marks)**

b. State and prove DeMorgan's laws. **(05 Marks)**

c. Find the number of integers from 1 to 200 that are (i) not divisible by 5, (ii) divisible by 2 or 5 or 9, iii) not divisible by 2 or 5 or 9. **(05 Marks)**

d. If the probability of hitting a target by three persons A, B, C are ½, ⅓, ¼ respectively find the probability that the target is hit by atleast one person. **(05 Marks)**

2 a. Define: i) tautology, ii) contradiction. Using truth table show that:

$[p \to (q \wedge r)] \Leftrightarrow [(p \to q) \wedge (p \to r)]$ **(07 Marks)**

b. Using the laws of logic (without truth table) prove that:

i) $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$, ii) $[p \vee q \vee (\neg p \wedge \neg q \wedge r)] \Leftrightarrow (p \vee q \vee r)$ **(06 Marks)**

c. Test the validity of argument: **(07 Marks)**

$$\begin{array}{l} p \to q \\ r \to s \\ \underline{p \vee r} \\ \therefore q \vee s \end{array}$$

3 a. Define open statement. For all integers if p(x) : x > 0; q(x) : x is even; r(x) : x is a perfect square s(x) : x is divisible by 3 then write down the following statements in symbolic form:

i) Atleast one integer is even

ii) There exists a positive integer that is even.

iii) Some even integers are divisible by 3.

iv) If x is even and a perfect square then x is not divisible by 3. **(07 Marks)**

b. Write down converse, inverse, contrapositive of, $\forall x, [p(x) \to q(x)]$. **(06 Marks)**

c. Give (i) a direct proof, (ii) an indirect proof and (iii) proof by contradiction, to the statement "If n is an even integer then n + 9 is an odd integer". **(07 Marks)**

4 a. Prove by mathematical induction, $n! \geq 2^{n-1}$ for all integers $n \geq 1$. **(06 Marks)**

b. Find the explicit form of a_n if the recursive relation is $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \geq 2$. **(07 Marks)**

c. Define Fibonacci sequence by stating its recursive relation. If $F_0 = 0$, $F_1 = 1$, find $F_2, F_3, F_4, F_5, F_6, F_7$. **(07 Marks)**

5 a. How many persons must be chosen in order that atleast five of them will have birth days in the same calendar month? **(05 Marks)**

b. Find the stirling number s(7, 4). If A = {1, 2, 3, 4, 5, 6, 7} and B = {a, b, c, d} how many onto functions from A to B exist. **(05 Marks)**

c. If $f : R \to R$, $g : R \to R$ are $f(x) = 5x + 6$, $g(x) = \frac{x-6}{5}$, prove that f and g are inverse of each other. **(05 Marks)**

5 d. If A = {, 2, 3, 4} with R is a relation on A given by x R y if and only if y = 2x then
   i) Write R as set of ordered pairs
   ii) Draw the digraph of R
   iii) Find the matrix of relation M_R.
   iv) Determine the in-degree and out-degree of vertices in the digraph of R. **(05 Marks)**

6 a. If A = {1, 2, 3, 4, 6, 12} with relation R defined as aRb iff a divides b. find R as a set of ordered pairs and prove that relation R is partial order on A. Also draw the Hasse diagram of the relation R. **(07 Marks)**

b. For the equivalence relation on A = {1, 2, 3, 4, 5} given by
   R = {(1, 1) (2, 2) (2, 3) (3, 2) (3, 3) (4, 4) (4, 5) (5, 4) (5, 5)},
   find the partition of a induced by R and show that it is a partition. **(06 Marks)**

c. For A = {1, 2, 3, 4, 5, 6, 7, 8} and partial order (A, R) given by Hasse diagram (Fig.Q6(c)) find all the lower bounds, upper bounds, greatest lower bounds (GLB) and least upper bound [LUB] of the subsets B_1 = {1, 2} and B_2 = {3, 4, 5}. **(07 Marks)**

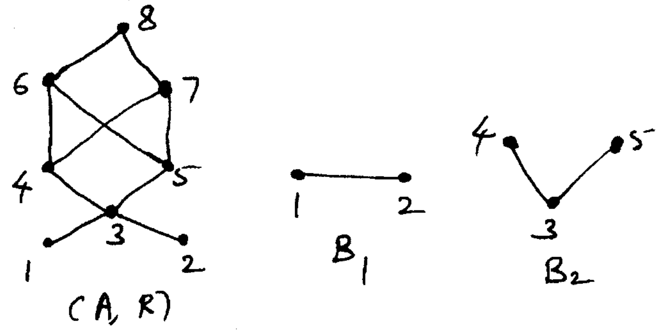

Fig.Q6(c)

7 a. If a 9-tupple say c = 011011101 is transmitted with p = 0.05 of incorrect transmission, find probability that (i) single error occurs, (ii) a double error occurs, (iii) a triple error occurs. **(07 Marks)**

b. The generator matrix of an encoding function $E: Z_2^2 \to Z_2^5$, $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$.
   i) Determine all the code words of Z_2^2.
   ii) Find the associated parity-check matrix H.
   iii) Decode 11101, 11011 using H. **(07 Marks)**

c. Prove that G = {1, -1, i, -i} is a cyclic group. **(03 Marks)**

d. If c = {00000, 01011, 10110, 11101} is a subgroup of Z_2^5 with Z_2 = {0, 1} under the addition modulo 2, using Lgrange's theorem find the number of cosets of C. **(03 Marks)**

8 a. If $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ is the generator matrix for an encoding function $E: Z_2^2 \to Z_2^5$,
   i) Find the group code C.
   ii) With x_1 = 10000, x_2 = 01000, x_3 = 00100, x_4 = 00010, x_5 = 00001, x_6 = 10100, x_7 = 10001, find cosets xi + c and form the decoding table
   iii) Using the above decoding table decode the received words 11111, 01111. **(12 Marks)**

b. Define Ring and give one example. **(04 Marks)**

c. Under the usual addition and multiplication of matrices show that set of all 2 × 2 matrices (M_2) is not Integral Domain. **(04 Marks)**

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